CALCULUS AB

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless other wise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_{\frac{\pi}{4}}^{x} \cos(2t)\,dt =$

(A) $\cos(2x)$ (B) $\frac{\sin(2x)-1}{2}$ (C) $\cos(2x)-1$ (D) $\sin(2x)$ (E) $\frac{\sin(2x)}{2}$

---

2. What are the coordinates of the point of inflection on the graph of $y = x^3 - 15x^2 + 33x + 100$?

(A) (9,0) (B) (5,–48) (C) (1,119) (D) (9,–89) (E) (5,15)

GO ON TO THE NEXT PAGE

3. If $3x^2 - 2xy + 3y = 1$, then when $x = 2$, $\frac{dy}{dx} =$

(A) -12 (B) -10 (C) $-\frac{10}{7}$ (D) 12 (E) 32

---

4. $\int_1^3 \frac{8}{x^3}\,dx =$

(A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$

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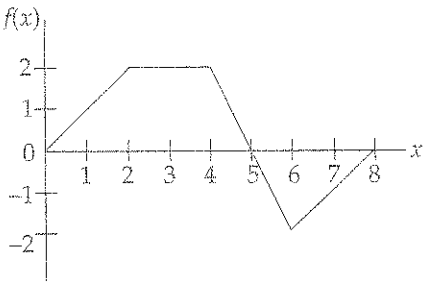
5. 

The graph of a piecewise linear function f, for $0 \le x \le 8$, is shown above. What is the value of $\int_0^8 f(x)\,dx$?

(A) 1 (B) 4 (C) 8 (D) 10 (E) 13

GO ON TO THE NEXT PAGE

6. If f is continuous for a $\leq x \leq b$, then at any point $x = c$, where $a < c < b$, which of the following must be true?

(A) $f(c) = \dfrac{f(b) - f(a)}{b - a}$

(B) $f(a) = f(b)$

(C) $f(c) = 0$

(D) $\displaystyle\int_a^b f(x)\,dx = f(c)$

(E) $\lim_{x \to c} f(x) = f(c)$

---

7. If $f(x) = x^2\sqrt{3x+1}$, then $f'(x) =$

(A) $\dfrac{-3x^2 - 2x}{\sqrt{3x+1}}$

(B) $\dfrac{9x^2 + 2x}{\sqrt{3x+1}}$

(C) $\dfrac{-9x^2 + 4x}{2\sqrt{3x+1}}$

(D) $\dfrac{15x^2 + 4x}{2\sqrt{3x+1}}$

(E) $\dfrac{-9x^2 - 4x}{2\sqrt{3x+1}}$

GO ON TO THE NEXT PAGE

8. What is the instantaneous rate of change at $t = -1$ of the function f, if $f(t) = \dfrac{t^3 + t}{4t + 1}$?

(A) $\dfrac{12}{9}$ (B) $\dfrac{4}{9}$ (C) $-\dfrac{20}{9}$ (D) $-\dfrac{4}{9}$ (E) $-\dfrac{12}{9}$

---

9. $\displaystyle\int_2^{e+1} \left(\frac{4}{x-1}\right) dx =$

(A) 4 (B) $4e$ (C) 0 (D) $-4e$ (E) -4

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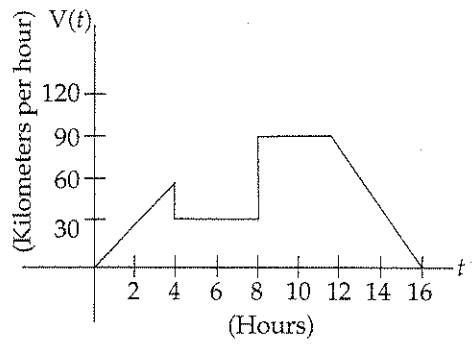
10. 

A car's velocity is shown on the graph above. Which of the following gives the total distance traveled from $t = 0$ to $t = 16$ (in kilometers)?

(A) 360 (B) 390 (C) 780 (D) 1000 (E) 1360

---

11. $\frac{d}{dx}\tan^2(4x) =$

(A) $8\tan(4x)$
(B) $4\sec^4(4x)$
(C) $8\tan(4x)\sec^2(4x)$
(D) $4\tan(4x)\sec^2(4x)$
(E) $8\sec^4(4x)$

GO ON TO THE NEXT PAGE

12. What is the equation of the line tangent to the graph of $y = \sin^2 x$ at $x = \frac{\pi}{4}$?

(A) $y - \frac{1}{2} = -\left(x - \frac{\pi}{4}\right)$

(B) $y - \frac{1}{2} = \left(x - \frac{\pi}{4}\right)$

(C) $y - \frac{1}{\sqrt{2}} = \left(x - \frac{\pi}{4}\right)$

(D) $y - \frac{1}{\sqrt{2}} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(E) $y - \frac{1}{2} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

---

13. If the function $f(x) = \begin{cases} 3ax^2 + 2bx + 1 \,;\, x \le 1 \\ ax^4 - 4bx^2 - 3x \,;\, x > 1 \end{cases}$ is differentiable for all real values of x, then $b =$

(A) $-\frac{11}{4}$ (B) $\frac{1}{4}$ (C) $-\frac{7}{16}$ (D) 0 (E) $-\frac{1}{4}$

GO ON TO THE NEXT PAGE

14. The graph of $y = x^4 + 8x^3 - 72x^2 + 4$ is concave down for

(A) $-6 < x < 2$

(B) $x > 2$

(C) $x < -6$

(D) $x < -3 - 3\sqrt{5}$ or $x > -3 + 3\sqrt{5}$

(E) $-3 - 3\sqrt{5} < x < -3 + 3\sqrt{5}$

---

15. If $f(x) = \dfrac{x^2 + 5x - 24}{x^2 + 10x + 16}$, then $\lim_{x \to -8} f(x)$ is

(A) 0 (B) 1 (C) $-\dfrac{3}{2}$ (D) $\dfrac{11}{6}$ (E) Nonexistent

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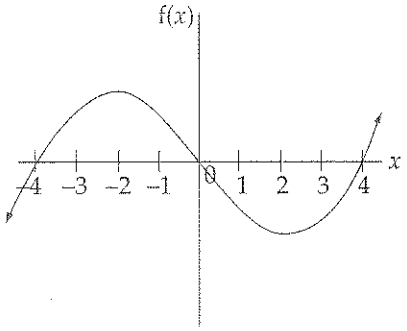
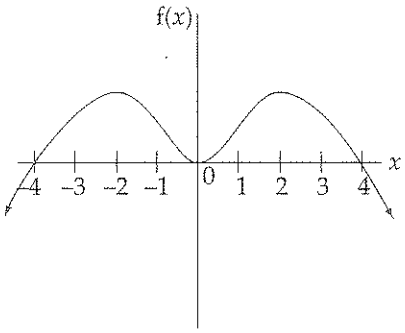
16.

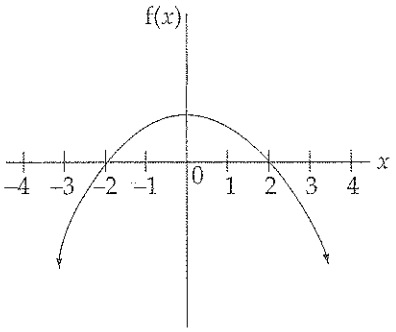


The graph of $f(x)$ is shown in the figure above. Which of the following could be the graph of $f'(x)$?

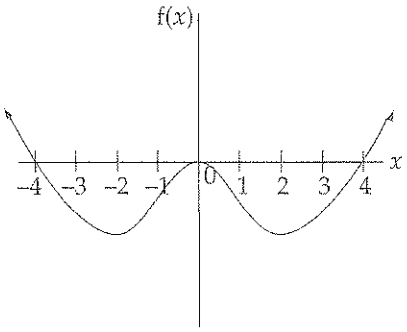
(A)



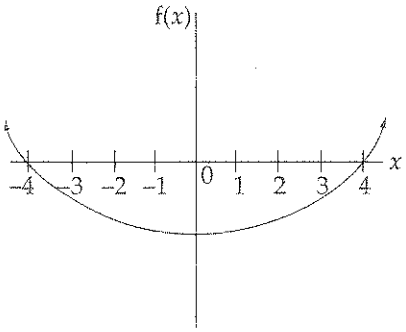
(B)



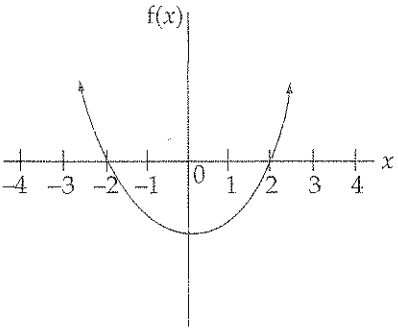
(C)



(D)



(E)



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17. If $f(x) = \ln(\cos(3x))$, then $f'(x) =$

(A) $-3\csc(3x)$
(B) $3\sec(3x)$
(C) $3\tan(3x)$
(D) $-3\tan(3x)$
(E) $-3\cot(3x)$

---

18. If $f(x) = \int_0^{x+1} \sqrt[3]{t^2 - 1}\, dt$, then $f'(-4)$

(A) $\sqrt[3]{-9}$ (B) -2 (C) 2 (D) $\sqrt[3]{15}$ (E) 0

---

19. A particle moves along the x-axis so that its position at time t, in seconds, is given by $x(t) = t^2 - 7t + 6$. For what value(s) of t is the velocity of the particle zero?

(A) 1 (B) 6 (C) 1 or 6 (D) 3.5 (E) 1 or 3.5 or 6

GO ON TO THE NEXT PAGE

20. $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\sin^2 x}\,dx =$

(A) e (B) $e-1$ (C) $1-e$ (D) $e+1$ (E) 1

---

21. The average value of $\sec^2 x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ is

(A) $\frac{8}{\pi}$

(B) $\frac{12\sqrt{3}-12}{\pi}$

(C) $\frac{12-4\sqrt{3}}{\pi}$

(D) $\frac{6\sqrt{2}-6}{\pi}$

(E) $\frac{6-6\sqrt{2}}{\pi}$

GO ON TO THE NEXT PAGE

22. Find the area of the region bounded by the parabolas $y = x^2$ and $y = 6x - x^2$.

(A) 9 (B) 27 (C) 6 (D) −9 (E) −18

---

23. The function f is given by $f(x) = x^4 + 4x^3$. On which of the following intervals is f decreasing?

(A) $(-3,0)$ (B) $(0,\infty)$ (C) $(-3,\infty)$ (D) $(-\infty,-3)$ (E) $(-\infty,0)$

---

24. $\lim_{x\to 0} \dfrac{\tan(3x)+3x}{\sin(5x)} =$

(A) 0 (B) $\frac{3}{5}$ (C) 1 (D) $\frac{6}{5}$ (E) Nonexistent

---

GO ON TO THE NEXT PAGE

25. If the region enclosed by the y-axis, the curve $y = 4\sqrt{x}$, and the line $y = 8$ is revolved about the x-axis, the volume of the solid generated is

(A) $\frac{32\pi}{3}$ (B) 128π (C) $\frac{128}{3}$ (D) 128 (E) $\frac{128\pi}{3}$

---

26. The maximum velocity attained on the interval $0 \le t \le 5$ by the particle whose displacement is given by $s(t) = 2t^3 - 12t^2 + 16t + 2$ is

(A) 286 (B) 46 (C) 16 (D) 0 (E) –8

---

27. The value of c that satisfies the Mean Value Theorem for Derivatives on the interval $[0,5]$ for the function $f(x) = x^3 - 6x$ is

(A) $-\frac{5}{\sqrt{3}}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) $\frac{5}{\sqrt{3}}$

GO ON TO THE NEXT PAGE

28. If $f(x) = \sec(4x)$, then $f'\left(\frac{\pi}{16}\right)$ is

(A) $4\sqrt{2}$ (B) $\sqrt{2}$ (C) 0 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{4}{\sqrt{2}}$

STOP

END OF PART A SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

CALCULUS AB

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The *exact* numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. If $f(x)$ is the function given by $f(x) = e^{3x} + 1$, at what value of x is the slope of the tangent line to $f(x)$ equal to 2?

(A) −.135 (B) 0 (C) .231 (D) −.366 (E) .693

---

30. The graph of the function $y = x^3 + 12x^2 + 15x + 3$ has a relative maximum at $x =$

(A) −10.613 (B) −.248 (C) −7.317 (D) −1.138 (E) −.683

GO ON TO THE NEXT PAGE

31. The side of a square is increasing at a constant rate of $0.4\ cm / sec$. In terms of the perimeter, P, what is the rate of change of the area of the square, in cm^2 / sec?

(A) $0.05P$ (B) $0.2P$ (C) $0.4P$ (D) $6.4P$ (E) $51.2P$

---

32. Let f be the function given by $f(x) = 3^x$. For what value of x is the slope of the line tangent to the curve at $(x, f(x))$ equal to 1?

(A) 1.099 (B) .086 (C) 0 (D) −.086 (E) −1.099

---

33. Given f and g are differentiable functions and

$$f(a) = -4,\ g(a) = c,\ g(c) = 10,\ f(c) = 15$$

$$f'(a) = 8,\ g'(a) = b,\ g'(c) = 5,\ f'(c) = 6$$

If $h(x) = f(g(x))$, find $h'(a)$

(A) $6b$ (B) $8b$ (C) $-4b$ (D) 80 (E) $15b$

GO ON TO THE NEXT PAGE

34. What is the area of the region in the first quadrant enclosed by the graph of $y = e^{\frac{x^2}{4}}$ and the line $y = 0.5$?

(A) 0.240 (B) 0.516 (C) 0.480 (D) 1.032 (E) 1.349

---

35. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

(A) 6.407 (B) 13.565 (C) 19.972 (D) 27.879 (E) 34.944

---

36. The second derivative of a function f is given by $f''(x) = x \sin x - 2$. How many points of inflection does f have on the interval $(-10,10)$?

(A) Zero (B) Two (C) Four (D) Six (E) Eight

GO ON TO THE NEXT PAGE

37. $\lim_{h\to 0} \dfrac{\sin\left(\dfrac{5\pi}{6}+h\right)-\dfrac{1}{2}}{h}$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) $-\frac{\sqrt{3}}{2}$

---

38. $\dfrac{d}{dx}\displaystyle\int_{2x}^{5x} \cos t\, dt =$

(A) $5\cos 5x - 2\cos 2x$

(B) $5\sin 5x - 2\sin 2x$

(C) $\cos 5x - \cos 2x$

(D) $\sin 5x - \sin 2x$

(E) $\frac{1}{5}\cos 5x - \frac{1}{2}\sin 2x$

---

39. The base of a solid S is the region enclosed by the graph of $4x + 5y = 20$, the x-axis, and the y-axis. If the cross-sections of S perpendicular to the x-axis are semicircles, then the volume of S is

(A) $\frac{5\pi}{3}$ (B) $\frac{10\pi}{3}$ (C) $\frac{50\pi}{3}$ (D) $\frac{225\pi}{3}$ (E) $\frac{425\pi}{3}$

GO ON TO THE NEXT PAGE

40. Which of the following is an equation of the line tangent to the graph of $y = x^3 + x^2$ at $y = 3$?

(A) $y = 33x - 63$

(B) $y = 33x - 135$

(C) $y = 6.488x - 1.175$

(D) $y = 6.488x - 4.620$

(E) $y = 6.488x - 10.620$

---

41. If $f'(x) = \ln x - x + 2$, at which of the following values of x does f have a relative minimum value?

(A) 5.146 (B) 3.146 (C) 1.000 (D) 0.159 (E) 0

---

42. Find the area of the region between the curve $y = \cos x$ and the x-axis from $x = 1$ to $x = 2$ radians.

(A) 0 (B) 0.068 (C) 0.249 (D) 1.751 (E) 2.592

GO ON TO THE NEXT PAGE

43. Let $f(x) = \int \cot x \, dx$; $0 < x < \pi$. If $f\left(\frac{\pi}{6}\right) = 1$, then $f(1) =$

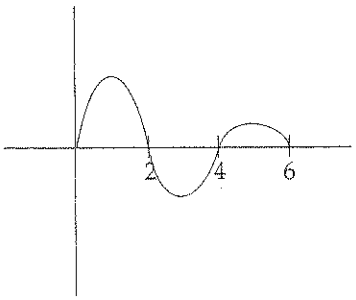
(A) –1.861 (B) –0.480 (C) 0.134 (D) 0.524 (E) 1.521

---

44. A radioactive isotope, y, decays according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in seconds. If the half-life of y is 1 minute, then the value of k is

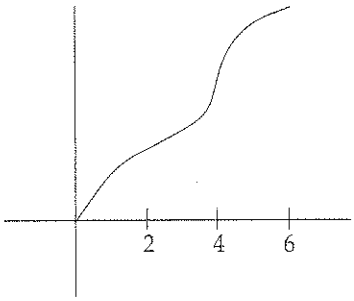
(A) –41.589 (B) –0.012 (C) 0.027 (D) 0.693 (E) 98.923

45.

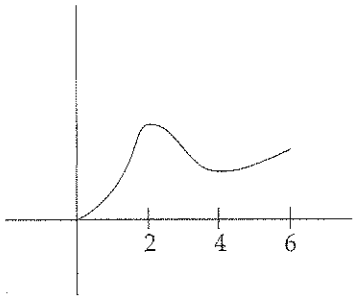


Let $g(x) = \int_0^x f(t)\,dt$, where $f(t)$ has the graph shown above. Which of the following could be the graph of g?

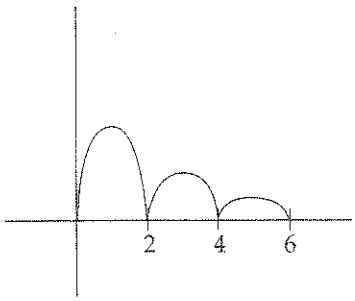
(A)



(B)



(C)



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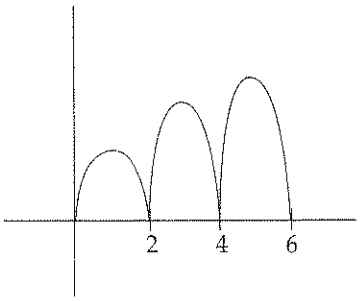
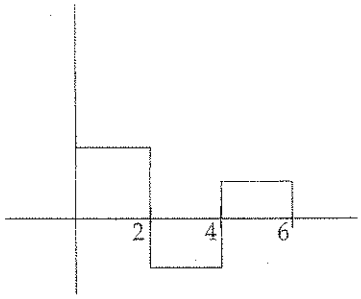


(E)



STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Percent of total grade—50

Part A: 45 minutes, 3 problems

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Part B: 45 minutes, 3 problems

(No calculator is allowed for these problems.)

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND SECTION II PART B

For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED

- Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly label functions, graphs, tables, or other objects you use.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If a calculation is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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CALCULUS AB

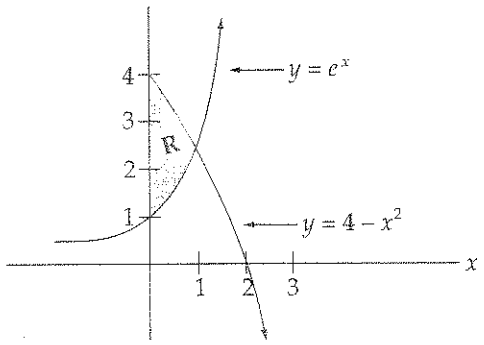
SECTION II, Part A

Time—45 Minutes

Number of questions—3

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

1.



Let R be the region in the first quadrant shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Find the volume of the solid generated when R is revolved about the line $x = -1$.

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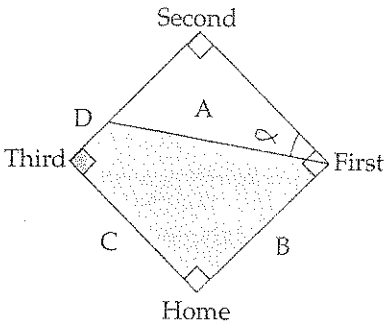
2.



A baseball diamond is a square with each side 90 feet in length. A player runs from second base to third base at a rate of 18 ft/sec.

(a) At what rate is the player's distance from first base, A, changing when his distance from third base, D, is 22.5 feet?

(b) At what rate is angle α increasing when D is 22.5 feet?

(c) At what rate is the area of the trapezoidal region, formed by line segments A, B, C, and D, changing when D is 22.5 feet?

GO ON TO THE NEXT PAGE

3. A body is coasting to a stop and the only force acting on it is a resistance proportional to its speed, according to the equation $\frac{ds}{dt} = v_f = v_0\, e^{-\left(\frac{k}{m}\right)t}$; $s(0) = 0$, where v_0 is the body's initial velocity (in *m/s*), v_f is its final velocity, m is its mass, k is a constant, and t is time.

(a) If a body with mass $m = 50kg$ and $k = 1.5kg/sec$, initially has a velocity of 30 *m/s*, how long, to the nearest second, will it take to slow to 1 *m/s*?

(b) How far, to the 10 nearest meters, will the body coast during the time it takes to slow from 30 *m/s* to 1 *m/s*?

c) If the body coasts from 30 *m/s* to a stop, how far will it coast?

CALCULUS AB

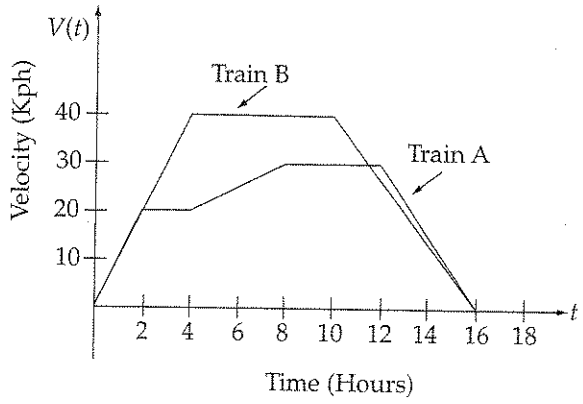
SECTION II, Part B

Time—45 Minutes

Number of questions—3

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS

4.



Three trains, A, B, and C each travel on a straight track for $0 \le t \le 16$ hours. The graphs above, which consist of line segments, show the velocities, in kilometers per hour, of trains A and B. The velocity of C is given by

$$v(t) = 8t - 0.25t^2$$

(Indicate units of measure for all answers.)

(a) Find the velocities of A and C at time $t = 6$ hours.

(b) Find the accelerations of B and C at time $t = 6$ hours.

(c) Find the positive difference between the total distance that A traveled and the total distance that B traveled in 16 hours.

(d) Find the total distance that C traveled in 16 hours.

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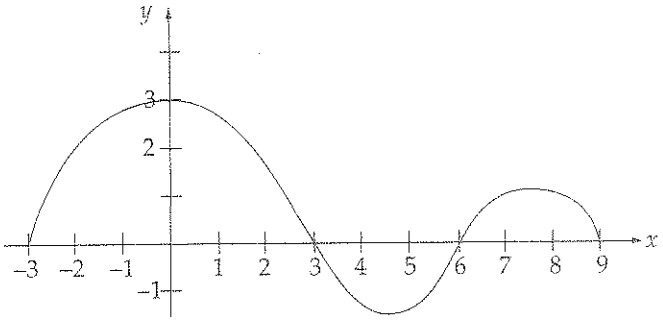
5.



The figure above shows the graph of $g(x)$, where g is the derivative of the function f, for $-3 \le x \le 9$. The graph consists of three semicircular regions and has horizontal tangent lines at $x = 0$, $x = 4.5$, and $x = 7.5$.

(a) Find all values of x, for $-3 < x \le 9$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-3 < x \le 9$, at which f attains a relative maximum. Justify your answer.

(c) If $f(x) = \int_{-3}^{x} g(t)dt$, find $f(6)$.

(d) Find all points where $f''(x) = 0$.

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6. Consider the curve given by $x^2y - 4x + y^2 = 2$.

(a) Find $\frac{dy}{dx}$.

(b) Find $\frac{d^2y}{dx^2}$.

(c) Find the equation of the tangent lines at each of the two points on the curve whose x-coordinate is 1.

END OF EXAMINATION